STAR POLYHEDRA: FROM ST MARK'S BASILICA IN VENICE TO HUNGARIAN PROTESTANT CHURCHES

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Summary

Star polyhedra appear on some baroque churches in Rome to represent papal heraldic symbols, and on the top of protestant churches in Hungary to represent the Star of Bethlehem. These star polyhedra occur in many different shapes. In this paper, we will provide a morphological overview of these star polyhedra, and we will reconstruct them by using the program package Mathematica 6.

Keywords: Star polyhedra; structural morphology; stellation; elevation; renaissance; baroque architecture; protestant churches; spire stars; Mathematica 6.0.

1. Introduction

Star is an ancient symbol that was important already for the Egyptians. Its geometrical representation has been a planar star polygon or a polygramma. In reliefs, they have got certain spatial character, but still they remained 2-dimensional objects. Real 3-dimensional representations of stars are *star polyhedra.* They are obtained by extending the faces of convex polyhedra so that their planes intersect again [1]. This technique is called *stellation*. In a broader sense, those polyhedra are also called star polyhedra that are obtained by *elevating* the face centres of the polyhedra, which are, in this way, augmented by pyramids (the base of a pyramid is identical to the face of the polyhedron on which it stands).

To our knowledge, star polyhedra do not occur in European culture until the Renaissance. The first example of a star polyhedron is a planar projection of a small stellated dodecahedron in the pavement mosaic of St Mark's Basilica in Venice (*Fig. 1a*). It is attributed to Paolo Uccello. Later, star polyhedra, together with other polyhedra, were studied by mathematicians and artists such as Luca Pacioli [2], Leonardo da Vinci (*Fig. 1b*), Wentzel Jamnitzer (*Fig. 1c*) and others [3]. Polyhedra, including also star polyhedra, as ornaments became popular since the beginning of the early baroque architecture in Italy. The star-on-hills symbol occurs in a number of papal coats of arms, which became a sculptural decoration on emphasized parts of the façade of buildings. This symbol and sometimes the star with a cross also appeared as a three-dimensional object on the façade or at the top of the dome of a church [4].

Star polyhedra occur often at the top of spires of protestant churches in Hungary [5,6]. These spire stars are symbols of the Star of Bethlehem. At the beginning, spire stars appeared as 2-dimensional polygons, but since the 19th century they have been produced as 3-dimensional polyhedra. Surprisingly, the shape of these spire stars shows a great variety.

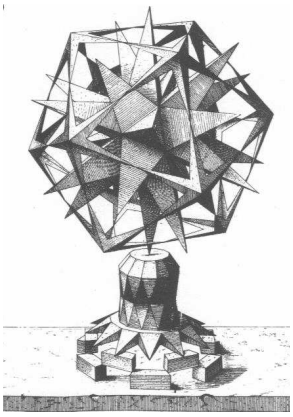
The aim of this paper is to provide an overview of the morphology of star polyhedra occurring on sacred buildings and, using the very recent version 6 of the Mathematica program package, to reconstruct the different star polyhedra. We want to show many types of these star polyhedra.

2. Star polyhedra in the baroque Rome

The first star polyhedra in Rome appeared during the pontificate of Pope Sixtus V (1585-1590). He had four obelisks re-erected: one in the Piazza San Pietro (1586), one in the Piazza dell'Esquilino (1587), one in the Piazza San Giovanni in Laterano (1588) and one in the Piazza del Popolo (1589) (*Fig. 2a*). The top of each obelisk is decorated with three mountains and a star, that is, with the main elements of his coat of arms.The heraldic star is represented by a compound

(a)

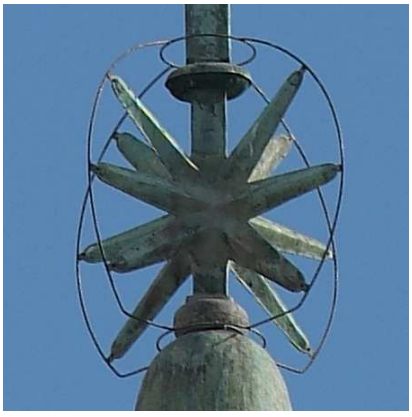
(b)



(c)

Fig. 1 (a) Small stellated dodecahedron in the pavement mosaic of St Mark's basilica, Venice, attributed to Paolo Uccello (1397-1475). (b) Elevated rhombicuboctahedron by Leonardo da Vinci (1452-1519). (c) Great stellated dodecahedron, etching by Wentzel Jamnitzer (1508-1585).

(a)



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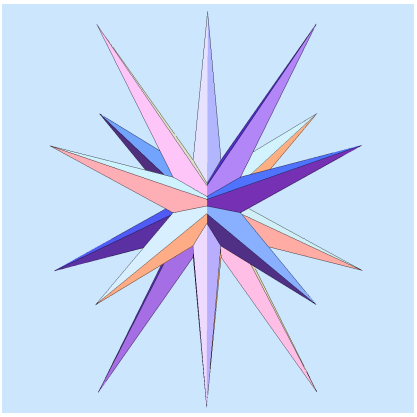
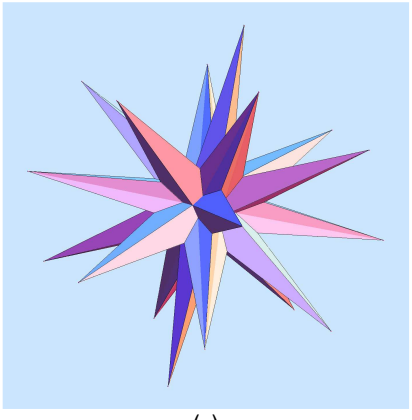


(c)

Fig. 2 (a) Obelisk in the Piazza del Popolo in Rome, re-erected by Pope Sixtus V in 1589. The top of the obelisk is decorated with his heraldic symbols: three mountains and a star. (b) The heraldic star is represented by a compound of two eight-pointed star dipyramids, (c) that is reconstructed by using Mathematica 6.

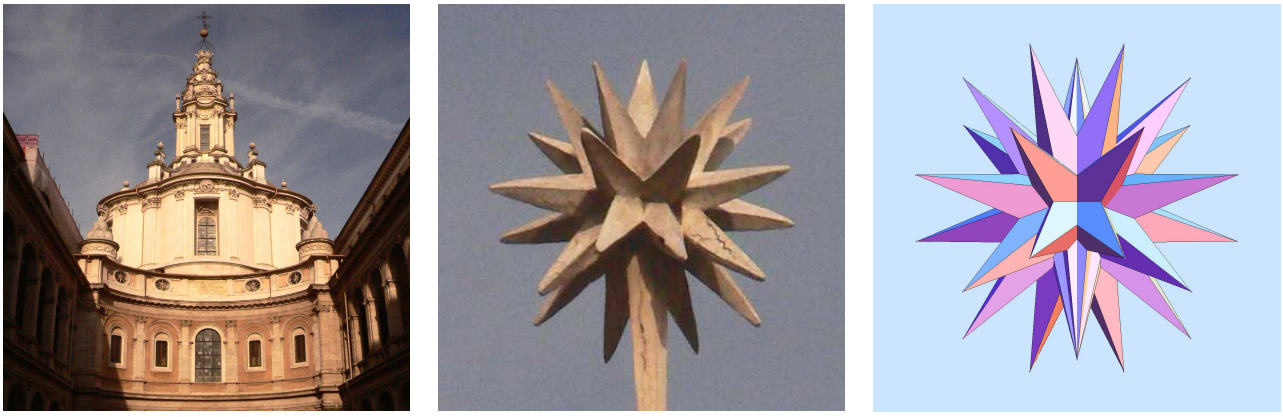
(a)

(b)



(c)

Fig. 3 (a) The church San Andrea della Valle in Rome. Its redesign was under Pope Alexander VII, by Carlo Rainaldi in 1665. (b) The star at the top of the dome is a compound of three eight-pointed star dipyramids, (c) that is reconstructed by using Mathematica 6.

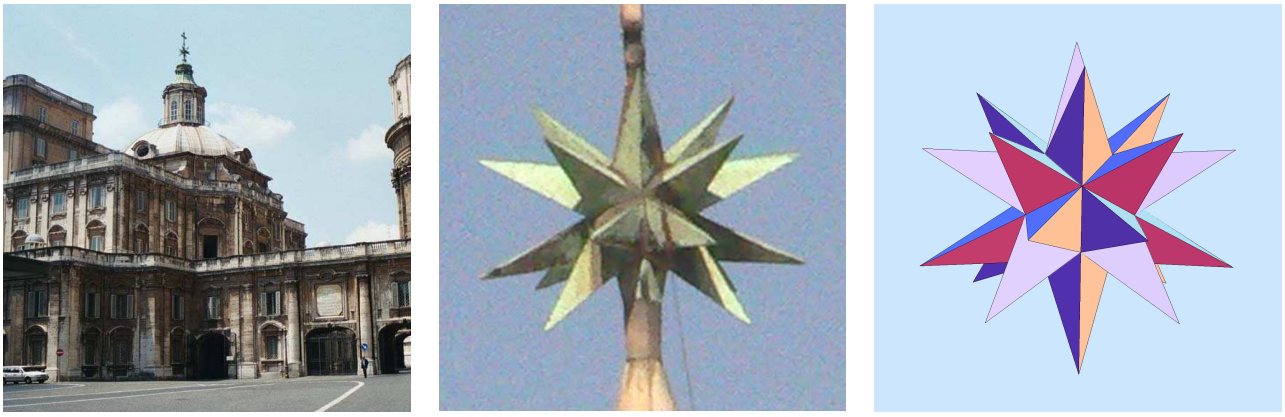


(a) (b) (c)

Fig. 4 (a) The church Sant'Ivo alla Sapienza in Rome, designed by Francesco Borromini, built 1642-1660. At the bottom of the drum of the dome, there are two copies of the heraldic symbols of Pope Alexander VII: six mountains and a star. (b) The heraldic star is represented by a compound of four eight-pointed star dipyramids, (c) that is reconstructed by using Mathematica 6.

(a) (b) (c)

Fig. 5 (a) The Archiginnasio della Sapienza in Rome, built by Pirro Ligorio and Giacomo della Porta, during the pontificates of Pope Gregorius XIII and Pope Sixtus V. The top of the tower is decorated with the main motifs of the coat of arms of Sixtus V: three mountains and a star. (b) The heraldic star is represented by an elevated rhombic dodecahedron, (c) reconstructed by Mathematica.



(a) (b) (c)

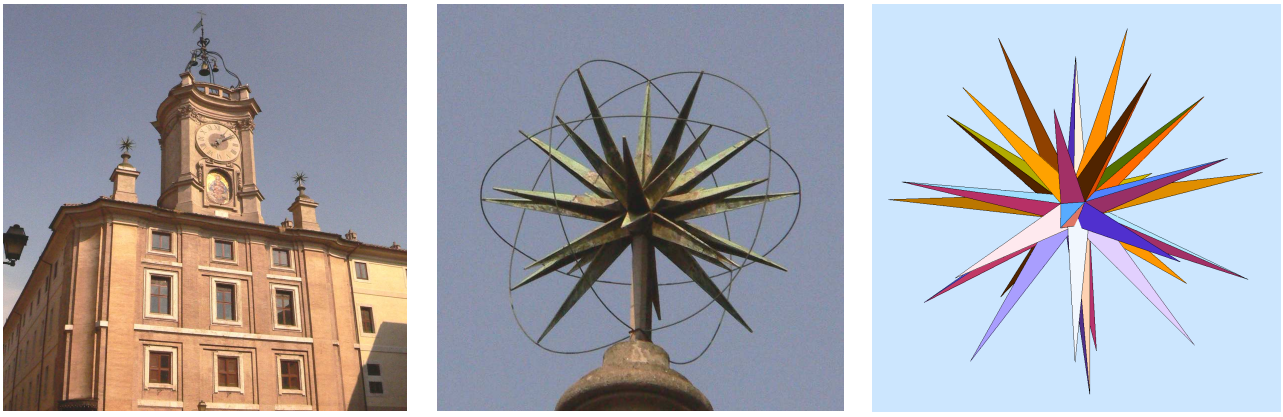
Fig. 6 (a) Sacristy of St Peter's in Rome, designed by Carlo Marchionni, built 1776-84. (b) The star in the coat of arms of Pope Pius VI appears at the top of the dome as a great stellated dodecahedron, (c) that is reconstructed by using Mathematica 6.

(a) (b) (c)

Fig. 7 (a) The Chigi chapel in the church Santa Maria del Popolo in Rome. The whole church was redesigned by Gian Lorenzo Bernini during the pontificate of Pope Alexander VII, and the top of the dome of the Chigi chapel was ornamented with the pope's symbols: six mountains and a star. (b) The star is represented by an elevated icosahedron, (c) reconstructed by Mathematica 6.

(a) (b) (c)

Fig. 8 (a) The church Santa Maria della Pace in Rome. Its façade was designed by Pietro da Cortona, built 1656-57, during the pontificate of Pope Alexander VII. The star of the coat of arms of the pope appears in three copies, two on the top of the façade and one on the portico. (b) Each star is represented by an elevated tri-augmented triangular prism, (c) reconstructed by using Mathematica 6.



(a) (b) (c)

Fig. 9 (a) The clock tower of Oratorio dei Filippini in Rome, designed by Francesco Borromini, built 1637-50. (b) It is decorated with two stars having the shape of an elevated di-augmented hexagonal antiprism with two additional points, (c) that is reconstructed by using Mathematica 6.

(*Fig. 2b*) obtained in the following way. Consider a planar 8-pointed star. Elevate the centre of the star polygon above and below the plane of the star to form a star dipyramid. Making two intersecting star dipyramids, by rotating a copy of the original by 90°, a 14-pointed compound is obtained. The tower of the Archiginnasio della Sapienza is also ornamented with the heraldic symbols of Pope Sixtus V (*Fig.5a*). The star here is an elevated rhombic dodecahedron (*Fig. 5b*). Six mountains and a star were the main elements of the coat of arms of the Chigi family and so also of Fabio Chigi who became Pope Alexander VII (1655-67). His heraldic star decorates the dome of the church San Andrea della Valle (*Fig. 3a*). The star appears in the form of a compound of three 8-pointed star dipyramids where the planes of the three star polygons are pair-wise perpendicular (*Fig. 3b*). This star compound has 18 points. The church Sant'Ivo alla Sapienza also bears his stars (*Fig. 4a*) which are constructed in the form of a compound of four 8-pointed star dipyramids (*Fig.4b*). This compound having 26 points is obtained by rotating three copies of the original star dipyramid by 45°, 90°, 135° about an axis passing through opposite points. The heraldic star of Alexander VII appears in the form of an elevated dodecahedron (small stellated dodecahedron with extended points) on the top of the obelisk erected in the Piazza di Santa Maria Sopra Minerva (1667). The dome of the Chigi chapel in the church Santa Maria del Popolo (*Fig. 7a*) is decorated with a star that is an elevated icosahedron (great stellated dodecahedron with extended points) (*Fig. 7b*). The elevated icosahedron was applied in three copies on the façade of the church San Andrea della Valle as well, and later on the top of more obelisks such as that in the Piazza della Rotonda (1711), Piazza del Quirinale (1786), Piazza della Trinita dei Monti (1789), Piazzale del Pincio (1822). A nice, almost perfect great stellated dodecahedron can be seen on the top of the Sacristy of St Peter's (*Fig. 6a,b*) constructed during the pontificate of Pope Pius VI (1775-99) [7]. Strange looking star polyhedra represent the heraldic star of Alexander VII on the church Santa Maria della Pace (*Fig. 8a,b*). These 14-pointed star polyhedra are obtained by elevation from a slightly modified Johnson solid the tri-augmented triangular prism [3]. *Figure 9* shows rather complicated 26-pointed star polyhedra. They stand on both sides of the clock tower of the Oratorio dei Filippini in the Piazza dell'Orologio. This star polyhedron can be constructed in the following way. Consider a hexagonal antiprism, and stand a pyramid on each of the two hexagonal faces such that the apexes of the pyramids are on the circumsphere of the antiprism. In this way, the antiprism is augmented with two hexagonal pyramids, and a convex polyhedron with 24 triangular faces is obtained. Then apply elevation and put two additional triangular pyramids at the apexes of the hexagonal pyramids.

Piperno [4] mentions further examples of star polyhedra in Rome and outside Rome, but we could not check and reconstruct them. These are on: the gate of the Villa Chigi in Rome, the main fountain in Villa Lante in Bagnaia, the fountain in the main square of Ariccia, the main fountain in Oriolo Romano, the obelisk in Urbino, and probably the church S.S. Annunziata in Genzano. Hargittai [7] also mentions star polyhedra on the gate in the Square of September 20 in Bologna.

3. Further star polyhedra

In addition to the star polyhedra in Rome and in its surroundings, we know some further examples of star polyhedra on other parts of the world. For instance, the iron fence of the Great Synagogue in Paris is ornamented with nice small stellated dodecahedra (*Fig. 10*), and the tower of the Mole Antonelliana in Turin is topped with a huge elevated dodecahedron (*Fig. 11*). There are star polyhedra also in China. The Exhibition Centre in Beijing (*Fig. 12*) was a present of the Soviet Union. Soviet architects often used five-pointed stars to decorate their buildings. That happened here, too. However, in addition to five-pointed star dipyramids, also star polyhedra decorate this building that is quite unusual. Unfortunately, we are unable to say what kind of star polyhedra was applied here. It came to our knowledge that the 19th century Evangelic church in Libcice nad Vltavou, Czech Republic and the 18th century town hall in Kluczbork, Poland are topped with a star polyhedron as well. Finally, it is worth mentioning that the known Christmas decoration called Moravian Star is also a star polyhedron. It is made with full or partial elevation of the rhombicuboctahedron (*Fig. 13*).

4. Star polyhedra in the architecture of protestant churches in Hungary

Star polyhedra as symbols played a special role in the architecture of protestant churches in Hungary. In the course of the 16th century, reformation became quite widespread in the country so that in a decade of the second half of the century, four fifth of the population followed Protestantism. The split between Lutherans and Calvinists and so the foundation of the Hungarian Reformed Church by the Calvinists can be dated back to the Synode of Debrecen held in 1567. This is also the period when the usage of distinctive symbols and signs of churches has become the custom [5]. A peculiar element of the architectural shape of the protestant churches is the pinnacle of the tower. The top point of spires, which has been ornamented with a cross at catholic churches, bore a star at protestant (Lutheran, Calvinist and

(a) (b) (c)

Fig. 10 (a) Grande Synagogue de Paris, rue de la Victoire, designed by Alfred Aldrophe, built 1867-74. Its iron fence is decorated with star polyhedra. (b) Each star polyhedron is a small stellated dodecahedron that has the property that, in certain views, the contour of the polyhedron is identical to the Star of David. (c) Reconstruction of the polyhedron by using Mathematica 6.

(a) (b) (c)

Fig. 11 (a) Mole Antonelliana, Turin, designed by Alessandro Antonelli, built for a synagogue during the years 1863-89. In 1904, a 4-m-diameter star was erected on the top of the building. (b) The star is an elevated dodecahedron (stellated small dodecahedron with extended points), (c) that is reconstructed by using Mathematica 6.

(a) (b)

Fig. 12 (a) The Exhibition Centre in Beijing, designed by Victor Andreiev, built in 1954. (b) It is decorated with five-pointed star dipyramids and with some star polyhedra. Unfortunately, we could not identify the structure of these star polyhedra.

(a) (b) (c)

Fig. 13 Moravian star invented by a teacher at Niesky, Germany, about 1850. It is an elevated rhombicuboctahedron, used for Christmas decoration. (a) The original design where all faces of the rhombicuboctahedron are elevated, resulting in a star with 26 points, (b) a lamp where no triangle faces are elevated, (c) the same as (b) but some points are extended more than the rest.

(a) (b) (c)

Fig. 14 (a) Lutheran church in the village of Irsa (today Albertirsa), Hungary. It was built in 1801, its spire with the star was completed in 1925 and renewed recently. (b) The old zinc spire star is an elevated rhombicuboctahedron where all faces are elevated. (c) Reconstruction of the star polyhedron by using Mathematica 6.

(a) (b) (c)

Fig. 15 (a) Reformed church in the town of Pécs, Hungary, designed by Imre Schlauch, built in 1907. (b) The spire star is a partially elevated rhombicuboctahedron where only the square faces are elevated. (c) Reconstruction of the star by using Mathematica 6.


(a)


(b)

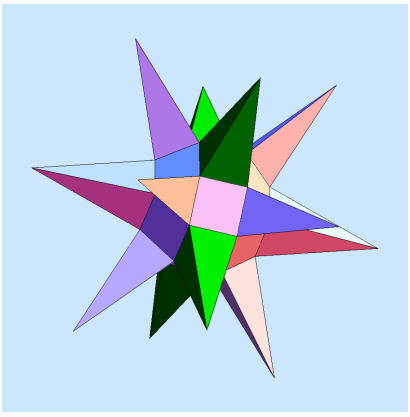

(c)

Fig. 16 (a) Reformed church in the town of Sopron, Hungary, designed by György Hárs, built in 1929. (b) The spire star is a partially elevated rhombicuboctahedron where only the faces corresponding to the faces and vertices of the parent cube are elevated (the square faces corresponding to the edges of the parent cube are not elevated). (c) Reconstruction of the star by using Mathematica 6.


(a)


(b)

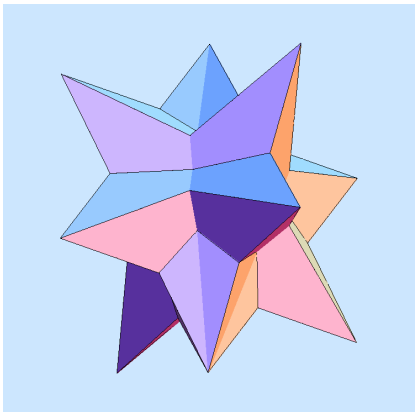

(c)

Fig. 17 (a) Lutheran church in the village of Felpéc, Hungary, built 1776-77. Its tower was completed in 1820. (b) The spire star is a compound of two six-pointed star dipyramids. There are views where the spire star looks like the Star of David. (c) Reconstruction of the star by using Mathematica 6.


(a)


(b)

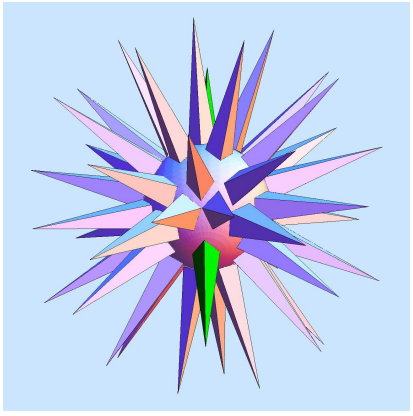

(c)

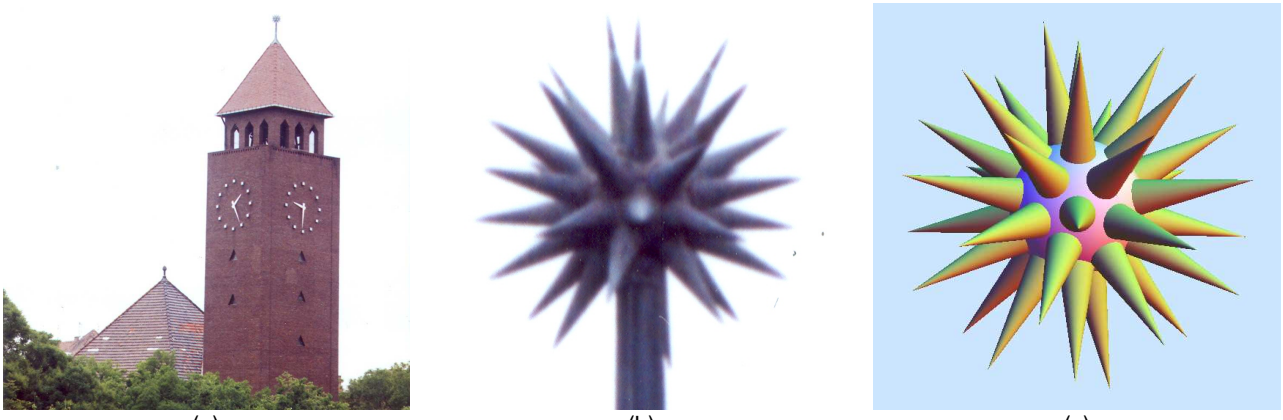
Fig. 18 (a) Reformed church in Kálvin Square, Budapest, Hungary, designed by Vince Hild and József Hofrichter, built 1816-30. Its spire was completed in 1859. (b) The spire star is composed of 1,10,10,10,10,1 equal rhombic pyramids packed along horizontal small circles on a sphere. (c) Reconstruction of the star by using Mathematica 6.

(a) (b) (c)

Fig. 19 (a) Reformed church in Nagyvárad Square, Budapest, Hungary, built 1930-35. (b) Its spire star is composed of 1,8,12,12,12,8,1 equal truncated square pyramids packed along horizontal small circles on a sphere. Pyramids in the three 12-member layers are aligned vertically. (c) Reconstruction of the spire star by using Mathematica 6.

(a) (b) (c)

Fig. 20 (a) Reformed church in Frangepán Sreet, Budapest, Hungary, built 1927-33. (b) The spire star is constructed of equal cones on a sphere such that the cones are placed at the centres of the faces of a rhombicuboctahedron. (c) Reconstruction of the spire star by using Mathematica 6.



(a) (b) (c)

Fig. 21 (a) Reformed church in the town of Szeged, Hungary, designed by József Boros, built 1941-47. (b) The spire star is composed of 1,4,8,8,8,4,1 equal cones packed along horizontal small circles on a sphere. (c) Reconstruction of the star by using Mathematica 6.

Unitarian) churches. The primary meaning of the star in the context of protestant art and architecture is the hint of the Star of Bethlehem as described in the Bible. The first examples of star symbols on spires of protestant churches had been realized as planar stars having a cast iron or copper structure but certainly there were stars made of wood which had been rapidly deteriorated and disappeared. The adequate shape of star polyhedra as a three-dimensional object appeared on church spires as a result of the development of copper and mainly the zinc industry in the 19th century. Catalogues of zinc factories offered a wide variety of tower spires with different symbols. Nowadays most Lutheran churches are topped with a cross and/or a star, while most reformed churches are topped with a star or a cock and never with a cross.

Figures 14-21 show a selection of many types of spire stars of Hungarian protestant churches, different from those in *Figs 2-11*. It is worth mentioning that stars in *Figs 20, 21* are not star polyhedra but configurations where equal circular cones are arranged on the surface of a sphere. In this way, these constructions are related to another part of geometry: packing of equal circles on a sphere [8].

5. Reconstruction of star polyhedra

The identified star polyhedra have been reconstructed by the new version 6 of the Wolfram Mathematica program package. Due to many advantages of the graphics package, in most cases, it was relatively easy to execute the elevation of the faces of the parent polyhedra and to find the required orientation. The reconstructed star polyhedra with the same orientation as that of the respective spire stars are presented in the subfigures *(c)* of *Figs 2-11* and *14-21*.

6. Conclusions

While walking in Rome or in Budapest people can find star polyhedra representing star symbols on sacred buildings. They are representatives of some dozen such stars occurring in Italy and of many hundred such stars occurring in Hungary. Their shape has not been studied, so far. But the shapes of these star polyhedra are not alike. We have discovered and shown in this paper how rich the world of forms of star polyhedra ornamenting buildings is, and identified and presented a number of different types of these star polyhedra.

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